Al-Adwaa Model 1

Algebra

1 Choose the correct answer:

(1) The simplest form of $n(x) = \frac{x+3}{x-3} \times \frac{x-3}{x^2-9}$ is ....

(a) $\frac{1}{x+3}$ (b) $\frac{1}{x-3}$ (c) $x+3$ (d) $x-3$

(2) If A and B are two mutually exclusive events, then P(A ∪ B) = ...

(a) P(B) (b) P(A ∩ B) (c) P(A) + P (B) (d) P(A)

(3) The set of zeroes of the function $f(x) = x(x^2 - 2x + 1)$ is ....

(a) {0 ,1} (b) {0, -1} (c) {0} (d) {1}

(4) The ordered pair which satisfies each of the following equations: $xy = 2$, $x - y = 1$ is ....

(a) (1, 2) (b) (2, 1) (c) (1 ,1) (d) (3, 1)

(5) The domain of the function $f: f(x) = \frac{2-x}{7}$ is ....

(a) $\mathbb{R} - \{7\}$ (b) $\mathbb{R} - \{2, 7\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R}$

(6) The domain of $n : n(x) = \frac{3x+4}{x^2+25} + \frac{x-2}{x^2+7}$ is ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5, 5, -7\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{-5, 5\}$

2

(a) Simplify: $n(x) = \frac{x}{x-2} \div \frac{x+3}{x^2-x-2}$,showing the domain of n

(b) Solve the following two equations in $\mathbb{R} \times \mathbb{R}$: $2x = 1 - y$, $x + 2y = 5$

3

(a) If A and B are two events of a random experiment and P(A) = 0.7 , P(A ∩ B) = 0.3,
find: P(A – B)

(b) Simplify: $n(x) = \frac{x^2+x}{x^2-1} - \frac{x+5}{x^2+4x-5}$,showing the domain of n

4

(a) Find in $\mathbb{R}$ the solution set of the following equation by using the general formula:
$x^2 - 4x + 1 = 0$ approximating the result to two decimal places.

(b) If $n_1(x) = \frac{2x}{2x+6}$, $n_2(x) = \frac{x^2+3x}{x^2+6x+9}$, **prove that:** $n_1 = n_2$:

5

(a) If $n(x) = \frac{x-2}{x+1}$

Find: (1) the domain of n^{-1} (2) $n^{-1}(3)$

(3) If $n^{-1}(x) = 2$, **find** the value of x.

(b) Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations: $x - y = 1$, $x^2 - y^2 = 25$

Al-Adwaa Model 2

1 Choose the correct answer:

(1) The set of zeroes of the function $f : f(x) = x(x^2 - 1)$ is ....

(a) $\{0\}$ (b) $\{0, -1\}$ (c) $\{0, 1, -1\}$ (d) $\{0, 1\}$

(2) The set of zeroes of the function $f : f(x) = \frac{x^2 - 9}{x - 2}$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\{-3, 3\}$ (c) $\{2\}$ (d) $\{3, -3, 2\}$

(3) If $n(x) = \frac{x - 2}{x + 5}$, then the domain of n^{-1} is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{2, -5\}$ (d) $\mathbb{R} - \{-5\}$

(4) The common domain of the two fractions $\frac{2}{x^2 - 1}$ and $\frac{5x}{x^2 - x}$ is ....

(a) $\mathbb{R} - \{1\}$ (b) $\mathbb{R} - \{0, 1\}$ (c) $\mathbb{R} - \{0, 1, -1\}$ (d) $\mathbb{R} - \{1, -1\}$

(5) The S.S. of the two equations: $x - y = 0$, $x^2 + y^2 = 18$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(3, 3)\}$ (b) $\{(-3, -3)\}$ (c) $\{(3, -3), (-3, 3)\}$ (d) $\{(3, 3), (-3, -3)\}$

(6) The set of zeroes of the function $f : f(x) = x^2 - 25$ is ....

(a) $\{5\}$ (b) $\{-5\}$ (c) $\{-5, 5\}$ (d) $\varnothing$

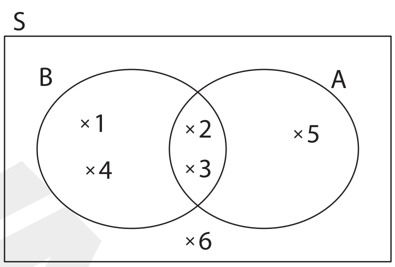
2

(a) If A and B are two events of a random experiment, then **find**:

(1) P (A ∩ B)

(2) P (A – B)

(3) The probability of non-occurrence of event A



(b) Simplify: $n(x) = \frac{x - 3}{x^2 - 7x + 12} - \frac{4}{x^2 - 4x}$, showing the domain of n.

3

(a) Find in $\mathbb{R}$ the solution set of the following equation by using the general rule:

$3x^2 - 5x - 4 = 0$ approximating the result to the nearest two decimal places.

(b) If the domain of the algebraic fraction $n : n(x) = \frac{x + 2}{x^2 + ax + b}$ is $\mathbb{R} - \{2, 3\}$.

Find the value of a and b.

4

(a) If $n(x) = \frac{x^2 - 3x}{(x - 3)(x^2 + 2)}$, then find $n^{-1}(x)$ in the simplest form, showing the domain of n^{-1}

(b) Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations: $x + y = 7$, $x^2 + y^2 = 25$

5

(a) Simplify: $n(x) = \frac{x^2 + 3x}{x^2 - 9} \div \frac{2x}{x + 3}$, showing the domain of n

(b) Solve the following two equations in $\mathbb{R} \times \mathbb{R}$: $x + 3y = 7$, $5x - y = 3$

Al-Adwaa Model 3

1 Choose the correct answer:

(1) If the S.S. of the two equations: $x + 2y = 5$ and $2x + ky = 3$ in $\mathbb{R} \times \mathbb{R}$ equals Ø, then k = ...

(a) 2 (b) –2 (c) 4 (d) –4

(2) Two positive numbers their sum is 9 and their product is 8, then the two numbers are ....

(a) 2,7 (b) 3,6 (c) 4 ,5 (d) 1,8

(3) If A is an event in the sample space of the random experiment, then P (A`) = ...

(a) 1 (b) –1 (c)1 – P (A) (d) P (A)–1

(4) If A and B are two events in a sample space for a random experiment $A \subset B$, then $P(A \cap B) =$ ....

(a) P (B) (b) P (A) (c) zero (d) ∅

(5) The solution set of the two equations: $x - y = 3$, $x + y = 7$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) {(6, 3)} (b) {(4 ,3)} (c) {(5, 2)} (d) {(3, 7)}

(6) If {3} is the solution set of the equation: $x^2 + mx = 3$, then m = ...

(a) –1 (b) –2 (c) 2 (d) 1

2

(a) Find in $\mathbb{R}$ the solution set of the following equation by using the general rule: $x^2 - 2x - 6 = 0$ approximating the result to one decimal places.

(b) A rectangle with a length more than its width by 4 cm if the perimeter of the rectangle is 28 cm, **find** the area of the rectangle.

3

(a) If the set of zeroes of the function $f : f(x) = ax^2 + x + b$ is $\{0, 1\}$ **find** the values of each two constants a and b.

(b) Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations: $y - x = 3$, $x^2 + y^2 - xy = 13$

4

(a) Simplify: $n(x) = \frac{x^3 - 8}{x^2 + x - 6} \times \frac{x + 3}{x^2 + 2x + 4}$,showing the domain of n

(b) Solve the following two equations in $\mathbb{R} \times \mathbb{R}$: $2x - y = 3$, $x + 2y = 4$

5

(a) If A and B are two events of a random experiment and P(A) = 0.3 , P(B) = 0.6 , $P(A \cap B) = 0.2$

Find: (1) $P(A \cup B)$ (2) P (A – B)

(b) Simplify: $n(x) = \frac{x^2 + 2x}{x^2 - 4} + \frac{x + 3}{x^2 - 5x + 6}$,showing the domain of n, then **find** $n(-2)$ if it is possible.

Al-Adwaa Model 1

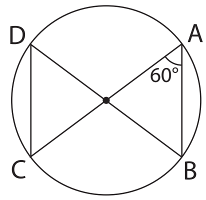
Geometry

1 Choose the correct answer:

(1) …………… is a line segment with one endpoint at the centre of the circle and the other endpoint on the circle.

(a) Diameter (b) Radius (c) Chord (d) Axis of symmetry

(2) In the opposite figure:



If m (∠ BAC) = 30°, then m (∠ BDC) = ……………

(a) 15° (b) 60°

(c) 30° (d) 90°

(3) The length of the arc opposite to a central angle of measure 30° in a circle of circumference 36 cm = …………… cm

(a) 18 (b) 9 (c) 4.5 (d) 3

(4) M and N are two circles of radii lengths 9 cm, and 4 cm respectively MN = 5 cm, then the two circles are ……………

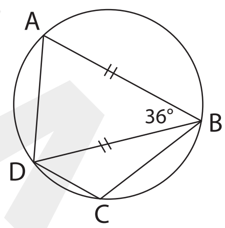
(a) touching externally (b) intersecting

(c) touching internally (d) distant

(5) The quadrilateral is cyclic if there is an exterior angle at any of its vertices …………… the measure of the interior angle at the opposite vertex.

(a) greater than (b) complements (c) supplements (d) equal to

(6) In the opposite figure:

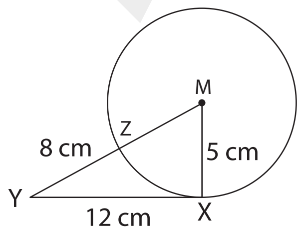


If AB = BD and m (∠ ABD) = 36°, then m (∠ C) = ……………

(a) 140° (b) 54°

(c) 70° (d)108°

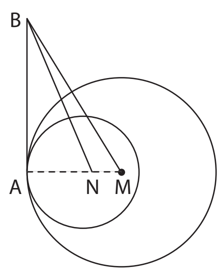
2 **(a) In the opposite figure:**



M is a circle with radius length 5 cm

XY = 12 cm, $\overline{MY}$ ∩ circle M = {Z} and ZY = 8 cm.

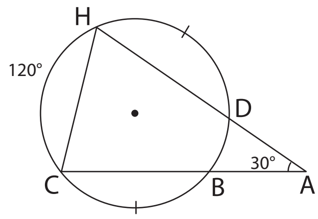
Prove that: $\overleftrightarrow{XY}$ is a tangent to circle M at X.



(b) M and N are two circles with radii lengths of 10 cm and 6 cm respectively and they are touching internally at A, $\overleftrightarrow{AB}$ is a common tangent for both at A.

If the area of the triangle BMN = 24 cm^2. **Find** the length of $\overline{AB}$.

3 **(a) In the opposite figure:**

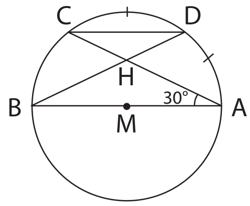


m (∠ A) = 30°, m $(\overset{\frown}{HC})$ = 120°, m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{DH})$

(1) **Find:** m $(\overset{\frown}{BD})$ the minor

(2) **Prove that:** AB = AD

(b) In the opposite figure:



$\overline{AB}$ is a diameter in the circle M

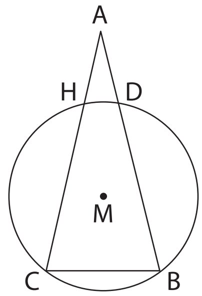
, C ∈ the circle M, m (∠ CAB) = 30°

, D is midpoint of $\overset{\frown}{AC}$, $\overline{DB}$ ∩ $\overline{AC}$ = {H}

(1) **Find:** m (∠ BDC) and m $(\overset{\frown}{AD})$

(2) **Prove that:** $\overline{AB}$ // $\overline{DC}$

4 **(a) In the opposite figure:**

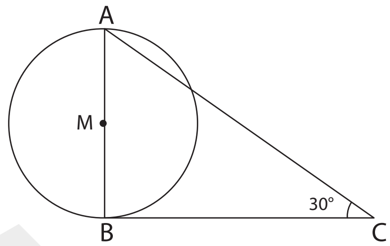


ABC is a triangle in which AB = AC , $\overline{BC}$ is a chord

in the circle M , if $\overline{AB}$ and $\overline{AC}$ cut the circle at D and H respectively.

Prove that: m $(\overset{\frown}{DB})$ = m $(\overset{\frown}{HC})$

(b) In the opposite figure:

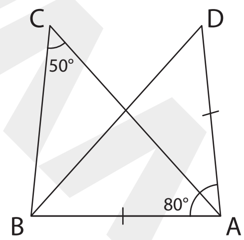


A circle M of circumference 44 cm, $\overline{AB}$ is a diameter,

$\overline{BC}$ is a tangent at B and m (∠ ACB) = 30°

Find the length of $\overline{BC}$ $\left(\pi = \frac{22}{7}\right)$

5 **(a) In the opposite figure:**



AB = AD , m (∠ DAB) = 80° , m (∠ C) = 50°

Prove that: The points A, B, C and D

have a circle passing through them.

(b) Mention two cases of the cyclic quadrilateral.

Al-Adwaa Model 2

1 Choose the correct answer:

(1) If M is a circle, its diameter length is 6 cm, and A is a point on the circle, then ……………

(a) MA > 6 cm (b) MA = 6 cm (c) MA = 3 cm (d) MA < 3 cm

(2) The type of the inscribed angle which is opposite to an arc greater than a semicircle is …………… angle.

(a) an acute (b) an obtuse (c) a right. (d) a straight

(3) $\overline{AB}$ and $\overline{CD}$ are two chords in a circle, AB = 5 cm and CD = 3 cm, then the chord which is nearer to the centre of the circle is ……………

(a) $\overline{AB}$ (b) $\overline{CD}$

(c) both are equal (d) cannot be determined

(4) We can identify the circle if we are given ……………

(a) three collinear points (b) two points

(c) three non-collinear points (d) one point

(5) If the measure of an angle of tangency = 70°, then the measure of the central angle subtended by the same arc equals ……………

(a) 35° (b) 70° (c) 140° (d) 105°

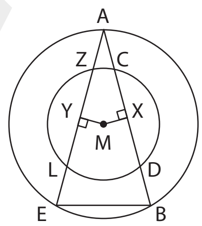
(6) The measure of the inscribed angle which is drawn in $\frac{1}{6}$ of a circle equals ……………

(a) 240° (b) 120° (c) 60° (d) 30°

2

(a) Two concentric circles M, $\overline{AB}$ is a chord in the larger circle and intersects the smaller circle at C and D, $\overline{AE}$ is a chord in the larger circle and intersects the smaller circle at Z and L, $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AE}$

If m (∠ ABE) = m (∠ AEB), then **prove that:** CD = ZL.

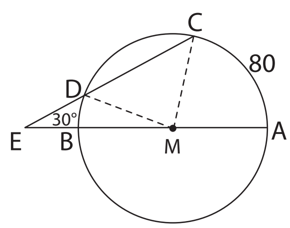


(b) In the opposite figure:

$\overline{AB}$ is a diameter in the circle M, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$,

m (∠ AEC) = 30°, m $(\overset{\frown}{AC})$ = 80°

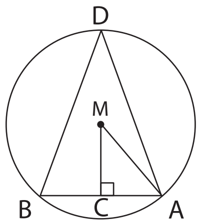
Find: m $(\overset{\frown}{CD})$



3 **(a) In the opposite figure:**

$\overline{AB}$ is a chord of circle M, $\overline{MC} \perp \overline{AB}$.

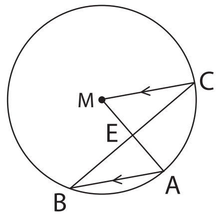
Prove that: m ($\angle$ AMC) = m ($\angle$ ADB)



(b) In the opposite figure:

$\overline{AB}$ is a chord in circle M, $\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$,

Prove that: BE > AE.

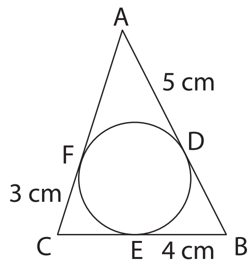


4 **(a) In the opposite figure:**

A circle is drawn touching the sides of a triangle ABC

, $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D, E, F, AD = 5 cm, BE = 4 cm, CF = 3 cm

Find the perimeter of Δ ABC

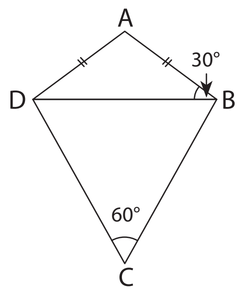


(b) In the opposite figure:

ABCD is a quadrilateral in which AB = AD,

m ($\angle$ ABD) = 30°, m ($\angle$ C) = 60°

Prove that: ABCD is a cyclic quadrilateral.



5 **(a) In the opposite figure:**

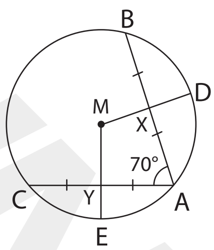
$\overline{AB}$ and $\overline{AC}$ are two chords equal in length at the circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m ($\angle$ A) = 70°

(1) **Find:** m ($\angle$ DME)

(2) **Prove that:** XD = YE



(b) In the opposite figure:

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

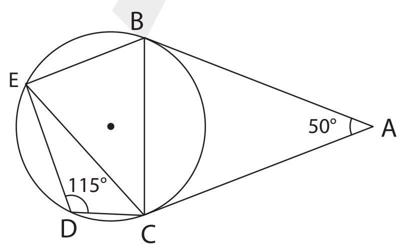
to the circle at B and C

, m ($\angle$ A) = 50° , m ($\angle$ D) = 115°

Prove that:

(1) $\overrightarrow{BC}$ bisects $\angle$ ABE

(2) CB = CE



Al-Adwaa Model 3

1 Choose the correct answer:

(1) If the chords of a circle are equal in length, then they are .... the centre.

(a) passing through (b) equidistant from

(c) intersecting at (d) perpendicular to

(2) The central angle whose measure is 90° subtended by an arc of length = ... the circumference of the circle.

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(3) The length of the arc that is opposite to a right inscribed angle in a circle whose circumference is 44 cm equals .... cm.

(a) 22 (b) 11 (c) $\frac{22}{7}$ (d) $\frac{44}{7}$

(4) Any straight line passing through the centre of the circle is called .... of it.

(a) diameter (b) radius (c) chord (d) axis of symmetry

(5) The number of common tangents of two non-congruent concentric circles is ....

(a) 1 (b) 2 (c) 4 (d) zero

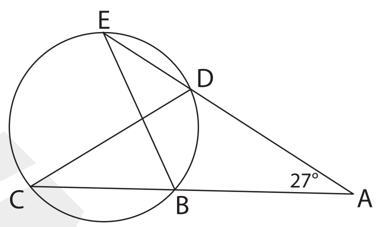
(6) In the opposite figure:

$\overrightarrow{AD}$ intersects the circle at D and E, $\overrightarrow{AB}$ intersects it at B and C.

If m (∠ A) = 27°, AB = BE, then m (∠ CDE) = ...

(a) 13.5° (b) 54°

(c) 27° (d) 36°

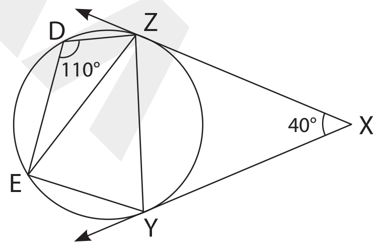


2 (a) In the opposite figure:

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle at the two point Y and Z, m (∠ X) = 40°

, m (∠ D) = 110°

Prove that: m (∠ ZYE) = m (∠ ZEY)

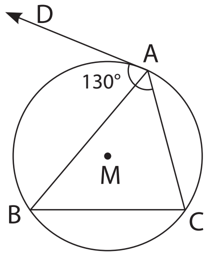


(b) In the opposite figure:

$\overrightarrow{AD}$ is the tangent to the circle M at A

, m (∠ DAC) = 130°

Find with proof: m (∠ B)

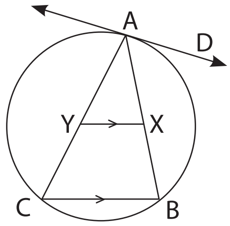


3 **(a)** ABC is a triangle inscribed in a circle,

$\overleftrightarrow{AD}$ is a tangent to the circle at A, X ∈ $\overline{AB}$, Y ∈ AC where $\overline{XY}$ // $\overline{BC}$

Prove that:

$\overline{AD}$ is a tangent to the circle passing through the points A, X and Y .



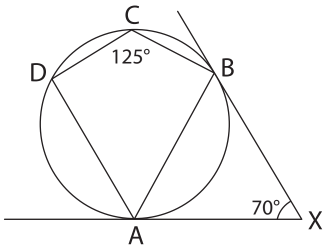
(b) $\overline{XA}$ and $\overline{XB}$ are two tangents to the circle at A and B

m (∠ AXB) = 70°, m (∠ DCB) = 125°

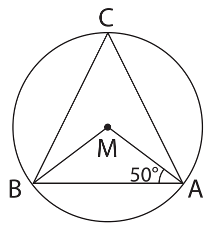
Prove that:

First: $\overrightarrow{AB}$ bisects ∠ DAX .

Second: $\overline{AD}$ // $\overline{XB}$.



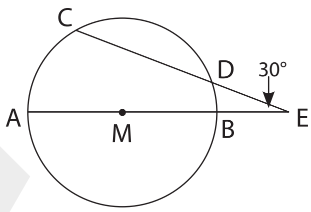
4 **(a)** M is a circle , m (∠ MAB) = 50° , find m (∠ C).



(b) In the opposite figure:

$\overline{AB}$ is a diameter in the circle M, $\overrightarrow{AB}$ ∩ $\overrightarrow{CD}$ = {E},

m (∠ E) = 30° , m ($\overset{\frown}{AC}$) = 80° , find m ($\overset{\frown}{BD}$)



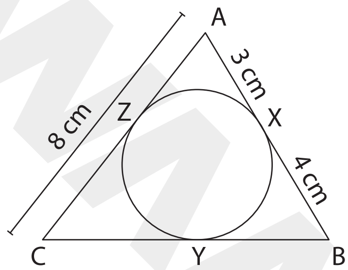
5 **(a) In the opposite figure:**

An inscribed circle of triangle ABC

touches its sides at X , Y and Z.

If AX = 3cm , XB = 4 cm , AC = 8 cm,

find the length of $\overline{BC}$.



(b) In the figure opposite:

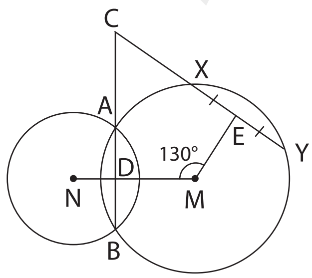
M and N are two intersecting circles

where circle M ∩ circle N equal {A , B}

$\overrightarrow{YC}$ ∩ $\overrightarrow{BA}$ = {C}

If E is the midpoint of $\overline{XY}$,

m (∠ EMN) = 130°, **find** m (∠ C).



Model Answer 1

Algebra

1 **Choose the correct answer:**

(1) $\frac{1}{x-3}$

(2) P (A) + P (B)

(3) $\{0, 1\}$

(4) (2 , 1)

(5) $\mathbb{R}$

(6) $\mathbb{R}$

2 **(a)** $n(x) = \frac{x}{x-2} \div \frac{x+3}{(x-2)(x+1)}$ factorize

Domain of $n = \mathbb{R} - \{2, -1, -3\}$

$n(x) = \frac{x}{x-2} \times \frac{(x-2)(x+1)}{x+3}$ switch to multiplication

$n(x) = \frac{x(x+1)}{x+3}$ simplify

(b) $y = (1 - 2x)$ (1)

$x + 2y = 5$ (2)

By substituting (1) in (2)

$x + 2(1 - 2x) = 5$

$x + 2 - 4x = 5$

$-3x = 3$

$x = -1$

Substitute in (1)

$y = (1 - 2(-1))$

$y = 3$

S.S. = $\{(-1, 3)\}$

3 **(a)** P(A – B) = P(A) – P(A ∩ B)

P(A – B) = 0.7 – 0.3 = 0.4

(b) $n(x) = \frac{x(x+1)}{(x-1)(x+1)} - \frac{x+5}{(x+5)(x-1)}$ factorize

Domain of n = $\mathbb{R} - \{1, -1, -5\}$

$n(x) = \frac{x}{(x-1)} - \frac{1}{(x-1)}$ simplify

$n(x) = \frac{x-1}{(x-1)}$ subtract

$n(x) = 1$ simplify

4 **(a)** a = 1 , b = –4 , c = 1

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$

$x = 2 + \sqrt{3} \simeq 3.73$, $x = 2 - \sqrt{3} \simeq 0.27$

S.S.= { 3.73 , 0.27 }

(b): $\because n_1(x) = \frac{2x}{2x+6} = \frac{2x}{2(x+3)} = \frac{x}{x+3}$

$\because$ The domain $n_1 = \mathbb{R} - \{-3\}$

$\because n_2(x) = \frac{x^2+3x}{x^2+6x+9} = \frac{x(x+3)}{(x+3)(x+3)} = \frac{x}{x+3}$

$\because$ The domain $n_2 = \mathbb{R} - \{-3\}$

$\because n_1(x) = n_2(x)$, domain of n_1 = domain of n_2

$\therefore n_1 = n_2$

5 **(a)** (1) $n(x) = \frac{x-2}{x+1}$

$n^{-1}(x) = \frac{x+1}{x-2}$

the domain of $n^{-1} = \mathbb{R} - \{-1 , 2\}$

(2) $n^{-1}(3) = \frac{3+1}{3-2} = 4$

(3) $\because n^{-1}(x) = \frac{x+1}{x-2}$

$\therefore \frac{x+1}{x-2} = 2$

$2(x-2) = x + 1$

$2x - 4 = x + 1$

$2x - x = 1 + 4$

$\therefore x = 5$

(b) $x - y = 1$ (1)

$x^2 - y^2 = 25$

$(x - y)(x + y) = 25$ (2)

Substitute (1) in (2)

$(x + y) = 25$ (3)

By adding (1) and (3)

Substitute (1) in (2)

$2x = 26$

$x = 13$

Substitute in (1)

$13 - y = 1$

$y = 12$ S.S = {(13, 12)}

Model Answer 2

1 Choose the correct answer:

(1) $\{0,1,-1\}$ (2) $\{-3, 3\}$ (3) $\mathbb{R} - \{2, -5\}$

(4) $\mathbb{R} - \{0,1,-1\}$ (5) $\{(3, 3), (-3, -3)\}$ (6) $\{-5, 5\}$

2 **(a)** (1) $P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

(2) $P(A - B) = \frac{1}{6}$

(3) $P(A`) = \frac{3}{6} = \frac{1}{2}$

(b) $n(x) = \frac{x-3}{(x-3)(x-4)} - \frac{4}{x(x-4)}$ factorize

Domain $n = \mathbb{R} - \{3, 4, 0\}$

$n(x) = \frac{1}{(x-4)} - \frac{4}{x(x-4)}$ simplify

$n(x) = \frac{x}{x(x-4)} - \frac{4}{x(x-4)}$ common denominator

$n(x) = \frac{x-4}{x(x-4)}$ subtract

$n(x) = \frac{1}{x}$ simplify

3 **(a)** $a = 3, b = -5, c = -4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-4)}}{2(3)}$

$x = \frac{5 \pm \sqrt{73}}{6}$

$x = \frac{5 + \sqrt{73}}{6} \simeq 2.26$, $x = \frac{5 - \sqrt{73}}{6} \simeq -0.59$

S.S. $= \{2.26, -0.59\}$

(b) $\because$ The domain of the $n(x)$ is $\mathbb{R} - \{2, 3\}$

$\therefore$ $n(2)$ and $n(3)$ are undefined

$\because n(2) = \frac{4}{2^2 + 2a + b}$

$4 - 2a + b = 0 \quad \therefore 2a + b = -4$ (1)

$\because n(3) = \frac{5}{3^2 + 3a + b}$ Type equation here.

$9 + 3a + b = 0 \quad \therefore 3a + b = -9$ (2)

$2a + b = -4$ $(x - 1)$

$3a + b = -9$

$-2a - b = 4$ (1)

$3a + b = -9$ (2) by adding (1) and (2)

$a = -5$ by substituting in the first equation $-10 + b = -4$

$b = 6$

4 **(a)** $n^{-1}(x) = \frac{(x-3)(x^2+2)}{x^2-3x} = \frac{(x-3)(x^2+2)}{x(x-3)}$

domain $= \mathbb{R} - \{0, 3\}$ $\quad n^{-1}(x) = \frac{(x^2+2)}{x}$

(b) $x = (7 - y)$ (1)

$x^2 + y^2 = 25$ (2)

Substitute (1) in (2)

$(7 - y)^2 + y^2 = 25$

$49 - 14y + y^2 + y^2 = 25$

$2y^2 - 14y + 49 = 25$

$2y^2 - 14y + 24 = 0$ $(\div 2)$

$y^2 - 7y + 12 = 0$

$(y - 4)(y - 3) = 0$

$y = 4$, $y = 3$ substitute in (1)

$x = 3$, $x = 4$

S.S. $= \{(3, 4), (4, 3)\}$

5 **(a)** $n(x) = \frac{x(x+3)}{(x+3)(x-3)} \div \frac{2x}{x+3}$ factorize

Domain of $n = \mathbb{R} - \{-3, 3, 0\}$

$n(x) = \frac{x(x+3)}{(x+3)(x-3)} \times \frac{x+3}{2x}$ switch to multiplication

$n(x) = \frac{x+3}{2(x-3)} = \frac{x+3}{2x-6}$ simplify

(b) $5x - y = 3$ $(\times 3)$

$15x - 3y = 9$ (1)

$x + 3y = 7$ (2)

By adding (1) , (2)

$16x = 16$

$x = 1$

Substitute in (2)

$1 + 3y = 7$

$3y = 6$

$y = 2$

S.S. $= \{(1,2)\}$

Model Answer 3

1 **Choose the correct answer:**

(1) 4

(2) 1,8

(3) 1 – P (A)

(4) P (A)

(5) {(5, 2)}

(6) –2

2 **(a)** $a = 1$, $b = -2$, $c = -6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)}$

$x = \frac{2 \pm \sqrt{28}}{2}$

$x = 1 + \sqrt{7} \simeq 3.6$, $x = 1 - \sqrt{7} \simeq -1.6$

S.S. = {3.6 , –1.6}

(b) Let the length = L and the width = W

$L = (W + 4)$ (1)

$2(L + W) = 28$ (2)

By substituting (1) in (2)

$2(W + 4 + W) = 28$

$2W + 4 = 14$

$2W = 10$

$W = 5$ cm

By substituting in (1)

$L = 5 + 4 = 9$ cm

Area = $L \times W = 9 \times 5 = 45$ cm^2

3 **(a)** $\because$ Z (f) = {0 , 1} by substituting $x = 0$

$\therefore$ b = 0

substituting $x = 0$

$a + 1 + 0 = 0$ $a = -1$

(b) $y = (x + 3)$ (1)

$x^2 + y^2 - xy = 13$ (2)

Substitute (1) in (2)

$x^2 + (x + 3)^2 - x(x + 3) = 13$

$x^2 + x^2 + 6x + 9 - x^2 - 3x - 13 = 0$

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

$x = -4 , x = 1$ substitute in (1)

$y = -1 , y = 4$

S.S. $= \{(-4 , -1) , (1 , 4)\}$

4 **(a)** $n(x) = \frac{(x-2)(x^2+2x+4)}{(x+3)(x-2)} \times \frac{x+3}{(x^2+2x+4)}$ factorize

Domain of $n = \mathbb{R} - \{-3 , 2\}$

$n(x) = 1$ simplify

(b): $2x - y = 3$ $(\times 2)$

$4x - 2y = 6$ (1)

$x + 2y = 4$ (2)

By adding (1) , (2)

$5x = 10$

$x = 2$

Substitute in (2)

$2 + 2y = 4$

$2y = 2$

$y = 1$

S.S. $= \{(2 , 1)\}$

5 **(a)** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.3 + 0.6 - 0.2 = 0.7$

$P(A - B) = P(A) - P(A \cap B)$

$P(A - B) = 0.3 - 0.2 = 0.1$

(b) $n(x) = \frac{x(x+2)}{(x+2)(x-2)} + \frac{x+3}{(x-3)(x-2)}$ factorize

Domain of $n = \mathbb{R} - \{3 , 2 , -2\}$

$n(x) = \frac{x}{(x-2)} + \frac{x+3}{(x-3)(x-2)}$ simplify

$n(x) = \frac{x(x-3)}{(x-2)(x-3)} + \frac{x+3}{(x-3)(x-2)}$ common denominator

$n(x) = \frac{x^2 - 3x + (x+3)}{(x-2)(x-3)}$ add

$n(x) = \frac{x^2 - 2x + 3}{(x-2)(x-3)} = \frac{x^2 - 2x + 3}{x^2 - 5x + 6}$

$n(-2)$ is undefined because $-2 \notin$ the domain

Model Answer 1

Geometry

1 **Choose the correct answer:**

(1) radius. (2) $60°$ (3) 3

(4) Touching internally (5) equal to (6) $108°$

2 **(a)** $\because \overline{MY} \cap$ circle $M = \{Z\}$

$\therefore MY = MZ + ZY$

$\because MZ = MX = 5$ cm (radii)

$\therefore MY = 5 + 8 = 13$ cm

$\because (MY)^2 = (13)^2 = 169$

$(MX)^2 = (5)^2 = 25$ $(XY)^2 = (12)^2 = 144$

$(MX)^2 + (XY)^2 = 25 + 144 + 169 = (MY)^2$

$\therefore m\angle MXY = 90°$ (The converse of the pythagoras' theorem)

$\because \overline{XY} \perp \overline{MX}$ and $\overline{MX}$ is a radius

$\therefore \overline{XY}$ is a tangent to the circle at X.

(b) $\because$ The two circles are touching internally at A

$\therefore A \in \overline{MN}$, $\overline{MN} \perp \overline{AB}$

$\because MN = 10 - 6 = 4$ cm (Touching internally)

$\because$ Area $\Delta BMN = \frac{1}{2} \times MN \times AB$

$\therefore 24 = 4 \times \frac{1}{2} \times AB$ $\therefore AB = 12$ cm

3 **(a)** $m(A) = \frac{1}{2}[m(\overset{\frown}{CH}) - m(\overset{\frown}{BD})]$

$30° = \frac{1}{2}[120 - m(\overset{\frown}{BD})]$

$60° = 120° - m(\overset{\frown}{BD})$

$m(\overset{\frown}{BD}) = 60°$

$m(\overset{\frown}{CH}) + m(\overset{\frown}{HD}) + m(\overset{\frown}{BD}) + m(\overset{\frown}{BC}) = 360°$

$\because m(\overset{\frown}{HD}) = m(\overset{\frown}{BC}) = \frac{360° - (120° + 60°)}{2}$

$m(\overset{\frown}{HD}) = m(\overset{\frown}{BC}) = 90°$

$\because \angle C$ is an inscribed angle subtended by $\overset{\frown}{HDB}$

$\because m(\angle C) = \frac{1}{2} m(\overset{\frown}{HDB}) = \frac{1}{2} \times 150° = 75°$

In Δ ACH:

$m(\angle H) = 180° - (30° + 75°) = 75°$

$m(\angle H) = m(\angle HCB) = 75°$

and $AH = AC$ (1)

$m(\overset{\frown}{BC}) = m(\overset{\frown}{HD})$

$HD = BC$ (2)

By subtracting (2) from (1)

$AH - AB = AC - BC$ $AD = AB$

(b) $\because \overline{AB}$ is a diameter

$\therefore$ m $(\overset{\frown}{AD})$ + m $(\overset{\frown}{CD})$ + m $(\overset{\frown}{BC})$ = 180°

m (∠ A) = 30°

$\because$ m $(\overset{\frown}{BC})$ = 60°

$\because$ m $(\overset{\frown}{CD})$ + m $(\overset{\frown}{AD})$ = 180° – 60 ° = 120°

$\therefore$ m $(\overset{\frown}{AD})$ = m $(\overset{\frown}{CD})$ = $\frac{120°}{2}$ = 60°

m ∠ ABD = $\frac{1}{2}$ m $(\overset{\frown}{AD})$= 30°

m ∠ CDB = $\frac{1}{2}$ m $(\overset{\frown}{BC})$= 30°

$\because$ m (∠ DBA) = m (∠ CDB) = 30° They are alternate

$\therefore \overline{AB} // \overline{CD}$

4 **(a)** $\because$ AB = AC

$\therefore$ m (∠B) = m (∠C)

$\therefore$ m $(\overset{\frown}{DHC})$ = m $(\overset{\frown}{HDB})$

by subtracting m $(\overset{\frown}{HD})$ from both sides

$\therefore$ m $(\overset{\frown}{DB})$ = m $(\overset{\frown}{HC})$

(b) $\because$ circumference = 44 cm

$\therefore 2\pi$ r = 44

r = 44 ÷ (2 × $\frac{22}{7}$) = 7 cm

$\because \overline{AB}$ is a diameter of length 14 cm and $\overline{BC}$ is a tangent

$\therefore$ m(∠B) = 90°

$\therefore$ m(∠ACB) = 30°

$\therefore$ AC = 28 cm and $(BC)^2 = (AC)^2 - (AB)^2$

BC = $\sqrt{28^2 - 14^2} = 14\sqrt{3}$ cm

5 **(a)** In Δ ABD

$\because$ AB = AD, m (∠DAB) = 80°

$\therefore$ m(∠D) = m(∠ABD) = $\frac{180° - 80°}{2}$ = 50°

$\because$ m(∠D) = m (∠C) = 50°

$\because$ They are both drawn angles on the same base $\overline{AB}$ and on one side of it.

$\because$ They points A, B , C and D have a circle passing through them.

(b) Mention any two cases of the following:

1-If there is a point in the plane equidistant from all vertices.

2- If there is an exterior angle its measure = the measure of the niterior angle at the opposite vertex.

3 - If there are two opposite angles are supplementary.

4- If there are two angles equal in measure and drawn on the same base and one side of this base.

Model Answer 2

1 Choose the correct answer:

(1) MA = 3 cm. (2) an obtuse (3) $\overline{AB}$

(4) three non-collinear points (5) 140° (6) 30°

2

(a) In Δ ABE:

∵ m (∠ ABE) = m (∠ AEB)

∴ AB = AE

In the larger circle:

∵ AB = AE

∵ $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AE}$

∴ MX = MY

In the smaller circle:

∵ MX = MY

∵ $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AE}$

∴ CD = ZL

(b) ∵ $\overline{AB}$ is a diameter in circle M

∴ m ($\overset{\frown}{ACB}$) =180°

Draw ($\overline{MC}$) , ($\overline{MD}$)

∵ m ($\overset{\frown}{AC}$) = 80°

∴ m (∠ AMC) = 80°

∴ m (∠ CME) = 180° – 80° = 100°

In triangle Δ CME:

∴ m (∠ ECM) = 180° – (30° + 100°) = 50°

In triangle Δ CMD:

∵ MC = MD radii

∴ m (∠ CMD) = 180° – (50° + 50°) = 80°

∴ m ($\overset{\frown}{CD}$) = 80°

3

(a) Draw $\overline{BM}$

Proof:

In Δ MAB:

∵ $\overline{MA} = \overline{MB}$ (radii), $\overline{MC} \perp \overline{AB}$

∴ m (∠ AMC) = m (∠ BMC) = $\frac{1}{2}$ m (∠ AMB) (1) isosceles triangle properties

∵ inscribed ∠ ADB and central ∠ AMB are subtended at ($\overset{\frown}{AB}$)

∴ m (∠ ADB) = $\frac{1}{2}$ m (∠ AMB) (2)

∴ From (1) and (2) we get: m (∠ AMC) = m (∠ ADB).

(b) $\because \overline{CM} // \overline{AB}$

$\therefore$ m (∠ CMA) = m (∠ MAB) alternate angles

$\because$ m (∠ CMA) = 2 × m (∠ CBA) central and inscribed

$\therefore$ m (∠ MAB) = 2 × m (∠ CBA)

$\therefore$ m (∠ MAB) > m (∠ CBA)

In Δ ABE: $\therefore$ BE > AE

4 **(a)** $\because \overline{AB}$, $\overline{BC}$ and $\overline{AD}$ are three tangents to the circle

$\therefore$ AD = AF = 5 cm.

BD = BE = 4 cm

CE = CF = 3 cm

$\because$ Perimeter of ⊿ABC = AB + BC + AC

$\therefore$ Perimeter of ⊿ABC = 8 + 9 + 7 = 24 cm.

(b) In ⊿ABC:

$\because$ AB = AD

$\therefore$ m (∠ ABD) = m (∠ MDB) = 30°

m (∠ BAD) = 180° – (30° + 30°) = 120°

m (∠ A) + m (∠ C) = 180° (opposite angles)

ABCD is a cyclic quadrilateral.

5 **(a) In the circle M:**

$\because$ X and y are midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

$\therefore \overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AC}$

And m (∠ MXA) = m (∠ MYA) = 90°

In the quadrilateral AXMY:

$\because$ m (∠ A) + m (∠ XMY) + m (∠ MXA) + m (∠ MYA) = 360°

$\therefore$ m (∠ DME) = 360° – (90 ° + 90 ° + 70°) = 110°

$\because$ AB = AC

$\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AC}$

$\therefore$ MX = MY $\because$ MD = ME = r

By subtracting MX from MD and MY from ME we get that: XD = YE

(b) $\overline{AB}$ and $\overline{AC}$ are two tangents

$\therefore$ AB = AC and m (∠ ABC) = m (∠ ACB) = $\frac{180° - 50°}{2}$ = 65°

$\because$ EBCD is a cyclic quadrilateral.

$\therefore$ m (∠ EDC) + m (∠ CBE) = 180°

m (∠ CBE) = 180° – 115° = 65°

$\because$ m (∠ ABC) = m (∠ EBC) = 65°

$\therefore \overrightarrow{BC}$ bisects ∠ ABE

$\because$ ∠ BEC is an inscribed angle subtended by ($\overset{\frown}{BC}$) and ∠ ABC is an angle of tangency subtended by ($\overset{\frown}{BC}$)

$\therefore$ m (∠ ABC) = m (∠ BEC) = 65°

$\because$ m (∠ EBC) = m (∠ BEC) = 65°

$\therefore$ CB = CE

Model Answer 3

1 Choose the correct answer:

(1) equidistant from	(2) $\frac{1}{4}$	(3) 22
(4) Axis of symmetry	(5) zero	(6) 54°

2 **(a)** $\because$ $\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents

$\therefore$ XY = XZ

And m ($\angle$ xzy) = m ($\angle$ xyz) = $\frac{180° - 40°}{2}$ = 70°

$\therefore$ ZYED is a cyclic quadrilateral.

$\therefore$ m ($\angle$ ZDE) + m ($\angle$ ZYE) = 180°

And m ($\angle$ EYZ) = 180° – 110° = 70° (1)

$\because$ $\angle$ ZEY is an inscribed angle subtended by ($\overset{\frown}{ZY}$)

and $\angle$ XZY is an angle of tangency subtended by ($\overset{\frown}{ZY}$)

$\therefore$ m ($\angle$ ZYE) = m ($\angle$ XZY) = 70° (2)

From (1) and (2) we get that:

m ($\angle$ ZYE) = m ($\angle$ ZEY)

(b) $\overrightarrow{AD}$ **is a tangent:**

$\therefore$ $\angle$ DAC is an angle of tangency subtended by ($\overset{\frown}{ABC}$)

and m ($\angle$ DAC)= $\frac{1}{2}$ m ($\overset{\frown}{ABC}$) =130°

And m ($\overset{\frown}{ABC}$) = 2 × 130° = 260°

$\because$ m ($\overset{\frown}{AC}$) + m ($\overset{\frown}{ABC}$) = 360°

$\therefore$ m ($\overset{\frown}{AC}$) = 360° – 260° = 100°

$\because$ $\angle$ B is an inscribed angle subtended by ($\overset{\frown}{AC}$)

$\therefore$ m ($\angle$ B) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$) = 50°

3 **(a)** $\because$ $\overleftrightarrow{AD}$ is a tangent and, $\overline{AB}$ is the chord of tangency.

$\therefore$ m ($\angle$ DAB) = m ($\angle$ C) (1) an inscribed angle and a central angle subtended by the same arc ($\overset{\frown}{AB}$)

$\because$ $\overline{XY}$ // $\overline{BC}$, $\overline{AC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ C) corresponding angle (2)

From (1) and (2) and we get: m ($\angle$ DAB) = m ($\angle$ AYX)

$\because$ m ($\angle$ DAX) = m ($\angle$ AYX)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A, X and Y

(b): Proof:

$\because$ $\overline{XA}$ and $\overline{XB}$ are two tangent segments.

$\therefore$ XA = XB

In Δ XAB:

$\because$ m ($\angle$ XAB) = m ($\angle$ XBA), m ($\angle$ X) = 70°

$\therefore$ m ($\angle$ XAB) = $\frac{180° - 70°}{2}$ = 55° (1)

$\because$ ABCD is a cyclic quadrilateral, m ($\angle$ C) = 125°

$\therefore$ m ($\angle$ DAB) = 180° – 125° = 55° (2)

From (1) and (2)

$\because$ m ($\angle$ XAB) = m ($\angle$ DAB) = 55°

$\therefore$ $\overline{AB}$ bisects $\angle$ DAX

$\because$ m ($\angle$ XBA) = m ($\angle$ DAB) = 55° alternate angles

$\therefore$ $\overline{AD}$ // $\overline{XB}$

4 **(a)** $\because$ MA = MB (radii)

$\therefore$ m ($\angle$ MAB) = m ($\angle$ MBA) = 50°

$\therefore$ m ($\angle$ AMB) = 180° – (50° + 50°) = 80°

$\therefore$ m ($\angle$ BCA) = 40° an inscribed angle and a central angle subtended by the arc ($\overset{\frown}{AB}$)

(b) $\because$ m ($\angle$ E) = 30° , m ($\overset{\frown}{AC}$) = 80°

$\therefore$ 30° = $\frac{180° - m(\overset{\frown}{BD})}{2}$

$\therefore$ m ($\overset{\frown}{BD}$) = 20°

5 **(a)** $\because$ The inscribed circle of the triangle ABC touches its sides at X , Y and Z

$\therefore$ AX = AZ = 3 cm , BX = BY = 4cm , CZ = CY

$\therefore$ CZ = 8 – 3 = 5 cm = CY

$\therefore$ BC = 4 + 5 = 9 cm

(b) $\because$ E is the midpoint of $\overline{XY}$

$\therefore$ $\overline{ME} \perp \overline{XY}$

$\because$ $\overline{AB}$ is a common chord of circles M , N

$\therefore$ $\overline{AB} \perp \overline{MN}$

$\because$ m ($\angle$ EMN) = 130°

$\therefore$ m ($\angle$ C) = 360 – (90 +90 + 130) = 50°